

Semi-transparent brane-worlds

Zoltán Keresztes¹, Ibolya Képiró², and László Á. Gergely¹

¹*Departments of Theoretical and Experimental Physics,
University of Szeged, 6720 Szeged, Dóm tér 9, Hungary*

²*Blackett Laboratory, Imperial College,
Prince Consort Road, London SW7 2BW, UK**

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Abstract

We study the evolution of a closed Friedmann brane perturbed by the Hawking radiation escaping a bulk black hole. The semi-transparent brane absorbes some of the infalling radiation, the rest being transmitted across the brane to the other bulk region. We characterize the cosmological evolution in terms of the transmission rate ε . For small values of ε a critical-like behaviour could be observed, when the acceleration due to radiation pressure and the deceleration induced by the increasing self-gravity of the brane roughly compensate each other, and cosmological evolution is approximately the same as without radiation. Lighter (heavier) branes than those with the critical energy density will recollapse slower (faster). This feature is obstructed at high values of ε , where the overall effect of the radiation is to speed-up the recollapse. We determine the maximal value of the transmission rate for which the critical-like behaviour is observed. We also study the effect of transmission on the evolution of different source terms of the Friedmann equation. We conclude that among all semi-transparent branes the slowest recollapse occurs for light branes with total absorption.

*Electronic address: zkeresztes@titan.physx.u-szeged.hu , ibolya.kepiro@imperial.ac.uk , gergely@physx.u-szeged.hu

I. INTRODUCTION

Enlarging the number of dimensions of space-time is a well established idea dating back to Kaluza and Klein. Originally such attempts were meant to incorporate the non-gravitational interactions into a geometric framework, and this could be partially achieved by adding non-compact spatial dimensions to the four-dimensional space-time.

A recent development in the subject was the suggestion to enrich the space-time by one non-compact dimension [1]. Starting from the assumption that gravitation is an effectively five-dimensional interaction, while the particles and fields of the standard model are effectively four-dimensional, we arrive to a novel theory of gravitation, known as brane-world gravity. In this theory our four-dimensional space-time is a hypersurface (the brane), embedded in the five-dimensional space-time (the bulk). In a generic setup both the brane and the bulk can be curved. The brane has a positive tension λ , which together with the 5-dimensional coupling constant $\tilde{\kappa}^2$ defines the four-dimensional coupling constant κ^2 as

$$6\kappa^2 = \tilde{\kappa}^4 \lambda. \quad (1)$$

The matter on the brane is induced by the embedding of the brane into the bulk. The Lanczos-Sen-Darmois-Israel junction conditions [2]-[5] relate the brane energy-momentum tensor T_{ab} (together with the brane tension) to the jump across the brane in the extrinsic curvature K_{ab} . In the simplest case, when the setup is symmetric; the extrinsic curvature on one side is exactly opposite to the extrinsic curvature on the other side. Thus the jump in the extrinsic curvature is simply twice the value of the extrinsic curvature on one side of the brane. However asymmetric embeddings were also considered [6]-[12]. Then the theory acquires interesting new features, like late-time acceleration in cosmological branes.

The bulk contains at least a negative cosmological constant $\tilde{\Lambda}$. In the simplest case $\tilde{\Lambda}$ can be fine-tuned to the brane tension such that the effective cosmological constant on the brane vanishes (Randall-Sundrum gauge, [1]). In a more generic setup $\tilde{\Lambda}$ and λ can combine in such a way, that the brane remains with a tiny cosmological constant Λ . In the most generic case, when we allow for both non-standard matter in the bulk (like a scalar field, possibly taking over the role of the bulk cosmological constant, or a radiation of quantum origin) and for asymmetric embedding, the brane cosmological constant is given by the relation

$$2\Lambda = \kappa^2 \lambda + \tilde{\kappa}^2 \tilde{\Lambda} - \frac{\bar{L}}{2} - \tilde{\kappa}^2 \overline{\tilde{T}_{cd} n^c n^d}, \quad (2)$$

where \tilde{T}_{cd} represents the energy-momentum tensor of the non-standard model bulk fields, n^a is the normal to the brane and \bar{L} is the trace of

$$\bar{L}_{ab} = \bar{K}_{ab}\bar{K} - \bar{K}_{ac}\bar{K}_b^c - \frac{g_{ab}}{2} \left(\bar{K}^2 - \bar{K}_{ab}\bar{K}^{ab} \right) . \quad (3)$$

An overbar denotes the average of a quantity, taken over the left and right sides of the brane. Therefore for a symmetric embedding $\bar{K}_{ab} = 0 = \bar{L}$. We remark that for either $\bar{L} \neq 0$ or $\overline{\tilde{T}_{cd}n^cn^d} \neq 0$, the quantity Λ defined by Eq. (2) can fail to be a constant [13].

On the brane, gravitational dynamics is modified as compared to general relativity. It is governed by a modified Einstein equation, derived in [13] as a generalization of the covariant approach of [14]:

$$G_{ab} = -\Lambda g_{ab} + \kappa^2 T_{ab} + \tilde{\kappa}^4 S_{ab} - \bar{\mathcal{E}}_{ab} + \bar{L}_{ab}^{TF} + \bar{\mathcal{P}}_{ab} , \quad (4)$$

where S_{ab} represents a quadratic expression in T_{ab} :

$$S_{ab} = \frac{1}{4} \left[-T_{ac}T_b^c + \frac{1}{3}T T_{ab} - \frac{g_{ab}}{2} \left(-T_{cd}T^{cd} + \frac{1}{3}T^2 \right) \right] , \quad (5)$$

and the source term $\bar{\mathcal{E}}_{ab}$ is the average of the electric part of the bulk Weyl tensor \tilde{C}_{abcd} , defined as

$$\mathcal{E}_{ac} = \tilde{C}_{abcd}n^bn^d . \quad (6)$$

The last source term is from the pull-back of the bulk sources to the brane

$$\bar{\mathcal{P}}_{ab} = \frac{2\tilde{\kappa}^2}{3} \overline{\left(\tilde{T}_{cd}g_a^cg_b^d \right)^{TF}} ,$$

and TF stands for trace-free. The source terms T_{ab} and S_{ab} are local, the latter modifying gravitational dynamics at high energies. By contrast, at low energies the effect of S_{ab} is negligible as compared to T_{ab} . Another feature is that the evolution of the source terms $-\bar{\mathcal{E}}_{ab} + \bar{L}_{ab}^{TF} + \bar{\mathcal{P}}_{ab}$ is non-local, as it depends on the bulk degrees of freedom. In order to close the system, the modified Einstein equation (4) has to be supplemented by the Codazzi equation and the twice contracted Gauss equation [13], as well as by evolution equations for the bulk fields.

When branes with cosmological symmetry are embedded into a Schwarzschild - anti de Sitter bulk (SAdS5), standard cosmological evolution emerges at low energies. By adding to the model an asymmetry in the bulk cosmological constant, a slight late-time acceleration

appears (see [15] and references therein). Cosmological branes embedded in a bulk containing radiation escaping from the brane were studied in [16], [17], [18], [19] and [20].

Other models, with Hawking radiation escaping from a bulk black hole, were also studied. These include branes with $k = 0$ [25] and closed brane-worlds [24]. In the latter model $k = 1$ was chosen because the expression for the energy density of the Hawking radiation was originally derived for closed universes [21]- [23]). Other essential features of the model discussed in [24] were the existence of only one bulk black hole, the radiation of which was completely absorbed on the brane. (By contrast, in the model [25], there were two bulk black holes and the radiation was completely transmitted.) The final conclusion of [24] was that the recollapsing fate of the $k = 1$ universe could not be avoided neither by the asymmetry due to the location of the single bulk black hole, nor by the absorbed bulk radiation. Indeed, the radiation contributed by two competing effects: a radiation pressure, accelerating the brane and an increase in the self-gravity of the brane (due to absorption), which contributed to a faster recollapse. For the case studied of total absorption on the brane a critical behaviour was observed for a certain value of the initial brane energy density, when these two effects roughly cancel each other, and cosmological evolution proceeds (roughly) as in the absence of radiation.

In this paper we introduce a new degree of freedom into the model of [24]. Specifically, we allow for the *transmission* across the brane. Thus the brane we study here is semi-transparent, in contrast with the completely opaque brane discussed in [24]. The rate of transmission is given by a parameter ε , zero for total absorption, and one for total transmission. We continue to neglect the reflection because there is no exact solution with cosmological constant describing a crossflow of radiation streams (although such a solution is known in four dimensions, in the absence of a cosmological constant [26]). According to these assumptions, the five-dimensional Vaidya-anti de Sitter (VAdS5) spacetime describes both bulk regions.

In Section 2 we develop the mathematics of our model. The radiation pressure accelerating the brane is obviously decreasing with increasing ε . The absorbed radiation, and the increase of self-gravity due to this is again decreasing with increasing ε . The question comes, how these two effects may change the cosmological evolution. Having set the relevant equations in dimensionless variables, we carry out a numerical analysis of the cosmological evolution in Section 3 and we present graphical solutions. We summarize our results on how

the semi-transparency of the brane affects the conclusions traced in [24] in the Concluding Remarks.

II. DYNAMICS OF THE SEMI-TRANSPARENT BRANE

When the brane obeys cosmological symmetries, for $k = 1$ it is described by the line element

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1+r^2} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \right]. \quad (7)$$

Here a denotes the scale factor. The modified Einstein equations (4) decouple into a generalized Friedmann and a generalized Raychaudhuri equation [13]. As shown in [13], the difference of the Codazzi equations, taken on the two sides of the brane gives the energy-balance equation. The last relevant equation is the energy emission of the bulk black hole.

We would like to write these equations in terms of dimensionless variables (these will be distinguished by a hat from the corresponding dimensional variables). Following [24], we define:

$$\begin{aligned} \hat{t} &= Ct, \quad \hat{a} = Ca, \quad \hat{H} = \frac{H}{C}, \\ \hat{\rho} &= \frac{\rho}{\lambda}, \quad \hat{p} = \frac{p}{\lambda}, \quad \hat{\psi} = \frac{\psi}{C\lambda}, \\ \widehat{\Delta m} &= C^2 \Delta m, \quad \widehat{\overline{m}} = C^2 \overline{m}. \end{aligned} \quad (8)$$

Here $C = \kappa\sqrt{\lambda}$, and units $c = \hbar = 1$ were chosen. H is the Hubble parameter, ρ is the brane energy density, p the pressure of the matter on the brane. The energy density of the radiation escaping the bulk black hole is denoted as ψ . The quantity Δm represents the difference of the mass *functions* of the VAdS5 metric and \overline{m} is their average.

The brane evolves cf. the energy balance, Friedmann- and Raychaudhuri equations [13]. The energy balance equation, written in dimensionless variables is

$$\hat{\rho}' + 3\hat{H}(\hat{\rho} + \hat{p}) = (1 - \varepsilon)\hat{\psi}, \quad (9)$$

the Raychaudhuri equation reads

$$\begin{aligned} \hat{H}' &= -\hat{H}^2 - \frac{1}{6}\hat{\rho}(1 + 2\hat{\rho}) - \frac{1}{2}\hat{p}(1 + \hat{\rho}) - \frac{2\widehat{\overline{m}}}{\hat{a}^4} \\ &\quad + \frac{9(\hat{p} - 1)(\widehat{\Delta m})^2}{2\hat{a}^8(\hat{\rho} + 1)^3} - \frac{(1 + \varepsilon)\hat{\psi}}{\sqrt{6}} + \frac{3(1 - \varepsilon)\hat{\psi}\widehat{\Delta m}}{\sqrt{6}\hat{a}^4(\hat{\rho} + 1)^2}, \end{aligned} \quad (10)$$

and the Friedmann equation is

$$\widehat{H}^2 = -\frac{1}{\widehat{a}^2} + \frac{\widehat{\rho}}{3} \left(1 + \frac{\widehat{\rho}}{2}\right) + \frac{2\widehat{m}}{\widehat{a}^4} + \frac{3(\widehat{\Delta m})^2}{2\widehat{a}^8(\widehat{\rho} + 1)^2}. \quad (11)$$

The energy density of the radiation escaping from the black hole, evaluated at the left side of the brane becomes [24]:

$$\widehat{\psi} = \frac{5\kappa^4\lambda\zeta(5)}{24\pi^9\widehat{a}^4\widehat{m}_L \left\{ \widehat{H} + (\widehat{\rho} + 1)/\sqrt{6} + 3\widehat{\Delta m}/[\sqrt{6}\widehat{a}^4(\widehat{\rho} + 1)] \right\}}, \quad (12)$$

where ζ is the Riemann-zeta function. Here $\widehat{m}_L = \widehat{m} - \widehat{\Delta m}/2$ is the value of the mass function near the left side of the brane. It contains both the mass of the black hole and the energy of the Hawking radiation. From (12) and the formulae (90), (98) and (99) of [13] employed for our model; by using the relations giving the embedding of the brane, we derive the evolutions of \widehat{m} and $\widehat{\Delta m}$:

$$\widehat{m}' = -\frac{\widehat{a}^4\widehat{\psi}}{\sqrt{6}} \left\{ (1 + \varepsilon) \left[\widehat{H} + \frac{3\widehat{\Delta m}}{\sqrt{6}\widehat{a}^4(\widehat{\rho} + 1)} \right] + (1 - \varepsilon) \frac{\widehat{\rho} + 1}{\sqrt{6}} \right\}, \quad (13)$$

$$\widehat{\Delta m}' = \frac{2\widehat{a}^4\widehat{\psi}}{\sqrt{6}} \left\{ (1 - \varepsilon) \left[\widehat{H} + \frac{3\widehat{\Delta m}}{\sqrt{6}\widehat{a}^4(\widehat{\rho} + 1)} \right] + (1 + \varepsilon) \frac{\widehat{\rho} + 1}{\sqrt{6}} \right\}. \quad (14)$$

Eqs. (9), (10), (11), (12), (13) and (14) give a closed, but complicated system of coupled differential equations, from which further information can be extracted only by a numerical study. Eqs. (13) and (14) give the evolutions of $\widehat{m}_L = \widehat{m} - \widehat{\Delta m}/2$ and $\widehat{m}_R = \widehat{m} + \widehat{\Delta m}/2$. The evolution of \widehat{m}_L separates from the system and it is integrable. The result is given by the formula (38) of [24], which is independent of the rate of transmission. This can happen because the evaporation of the bulk black hole does not depend on the transmission rate across the brane.

III. COSMOLOGICAL EVOLUTION OF THE SEMI-TRANSPARENT BRANE

The partial transmission across the brane induces new features in the model. First, the accumulated energy from the absorbed Hawking radiation on the brane will be smaller than in the case of total absorption. Second, the transmitted radiation does not contribute to the radiation pressure on the brane, which also becomes smaller. We also note, that without

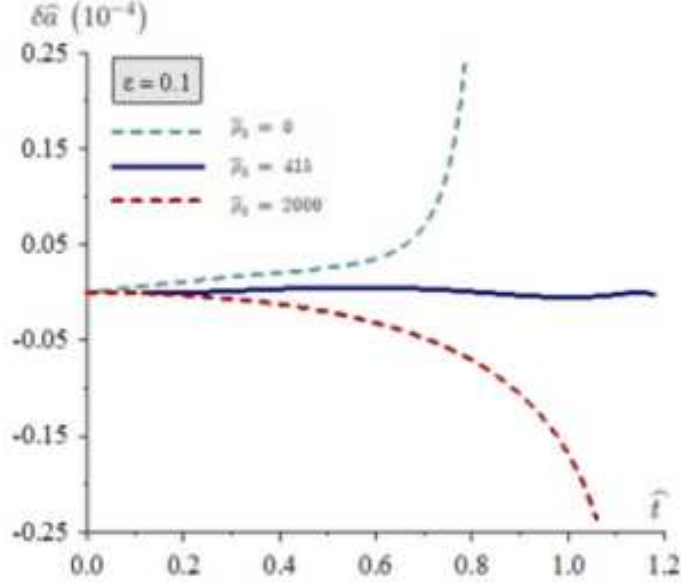


FIG. 1: Time evolution of the difference $\delta\hat{a}$ between the (dimensionless) scale factors in the radiating and non-radiating cases. For the depicted transmission rate of $\varepsilon = 0.1$ the critical-like behaviour $\delta\hat{a} \approx 0$ is observed at $\hat{\rho}_0^{crit} = 415$ (solid curve). This critical value of the initial energy density is smaller than for $\varepsilon = 0$ (then $\hat{\rho}_0^{crit} = 520$ was observed, see [24]). For small initial energy densities (in the extreme case $\hat{\rho}_0 = 0$, upper dashed curve), the pressure of the Hawking radiation dominates. For high initial energy densities (like $\hat{\rho}_0 = 2000$, lower dashed curve) the increase of self-gravity due to absorption overtakes the radiation pressure.

transmission and with only one black hole in the bulk, one of the bulk regions was SAdS5 in [24], as opposed to the present case, when both regions are VAdS5.

For the completely opaque branes discussed in [24], both the acceleration from the radiation pressure and the deceleration from the increase in the self-gravity of the brane were small perturbations of order 10^{-4} , which roughly cancelled each other for the critical initial energy density. As the semi-transparency of the branes further diminishes both effects of the radiation, the perturbations will be even smaller here, although of the same order of magnitude. We again plot only the *differences* of the physical quantities characterizing cosmological evolution, taken in the radiating and non-radiating cases.

As in [24], we take a radiation-dominated brane (with $\hat{p} = \hat{\rho}/3$) and start the evolution of the brane at the apparent horizon of the bulk black hole. For more details on the chosen initial data see [24].

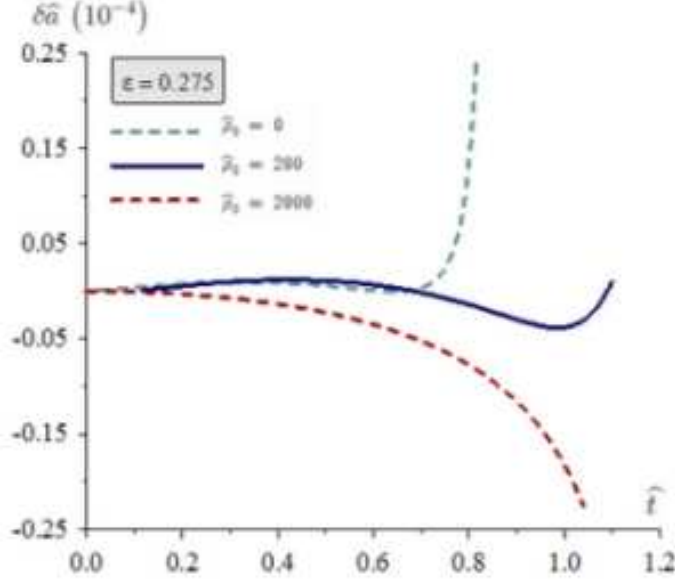


FIG. 2: As in Fig. 1, but for a higher transmission rate $\varepsilon = 0.275$. The *critical* brane initial energy density further decreases to $\hat{\rho}_0^{crit} = 200$. Moreover, the amplitude of the sinusoidal evolution of $\delta\hat{a}$ on the critical curve increases, damping the critical-like behaviour as compared to smaller absorption rates.

Figures 1-3 show the evolution of the differences in the scale factor when the radiation is switched on and off, for increasing transmission rates. Their sequence shows that the critical-like behaviour is seriously deteriorated by increasing transmission across the brane. For tiny values of ε we have found that the value of the critical energy density decreases with increasing transmission rate. Also the sinusoidal-like pattern of the critical curve is more accentuated, as the transmission increases (Fig. 1-2). The highest value of the transmission rate, for which such a critical-like behaviour could be observed is for $\varepsilon = 0.275$ (Fig. 2). For $\varepsilon > 0.275$, there is no value of the initial energy density for which the scale factor is larger in the presence of the radiation than in the non-radiating case. This holds true during the whole cosmological evolution. Therefore no critical-like evolution of the perturbation in the scale factor can be observed for these higher values of the transmission (Fig 3).

For total transmission, there is no radiation pressure on the brane and no increase in its self-gravity due to absorption at all. In this case, the Hawking radiation appears only in the Raychaudhuri equation through the term $-(1 + \varepsilon)\hat{\psi}/\sqrt{6}$. This term contributes toward deceleration, thus driving the universe towards a faster recollapse. In consequence the scale

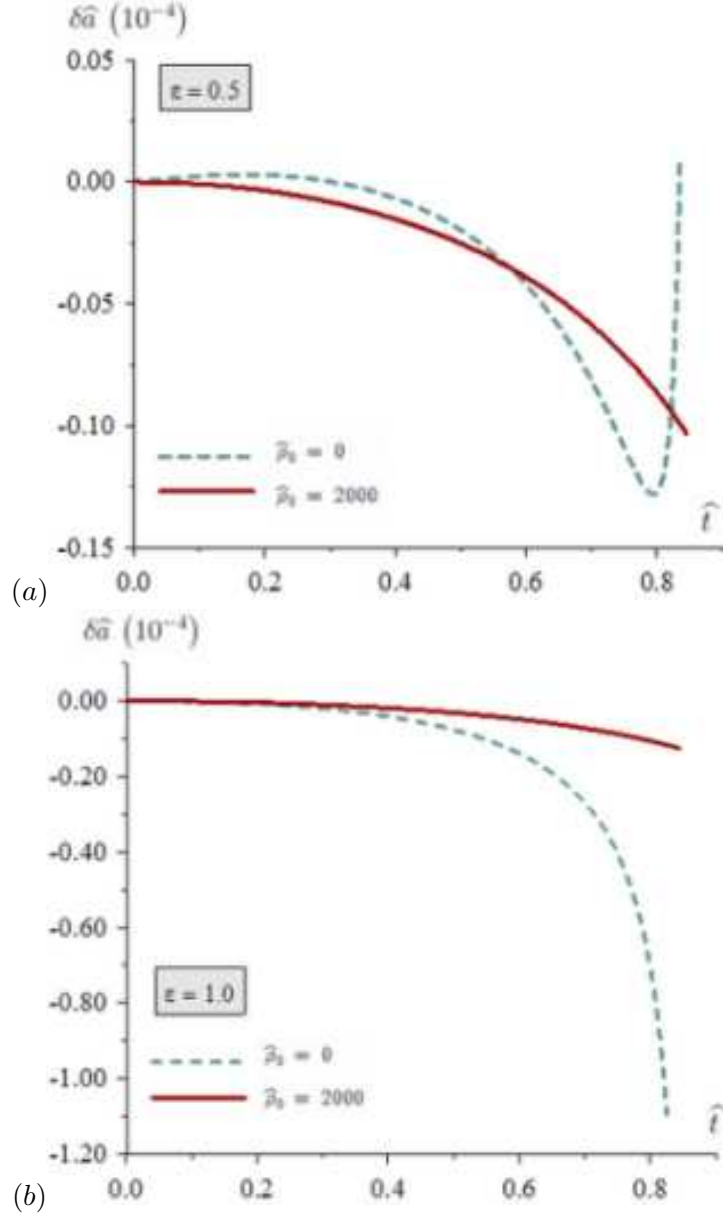


FIG. 3: For even higher transmission rates, there is no critical-like brane evolution at all. Therefore the time evolution of the difference $\delta\hat{a}$ between the (dimensionless) scale factors in the radiating and non-radiating cases is depicted only for the initial values $\hat{\rho}_0 = 0$ (solid curve) and $\hat{\rho}_0 = 2000$ (dashed curve). For any value of the initial energy density the net contribution of the radiation is towards a faster recollapse. The cases shown are (a) $\varepsilon = 0.5$ (b) total transmission $\varepsilon = 1$.

factor is smaller than in the non-radiating case (Fig. 3b).

Fig. 4 shows the time evolution of the difference between the brane energy densities in the radiating and non-radiating cases, plotted for $\hat{\rho}_0 = 300$ and various transmission rates.

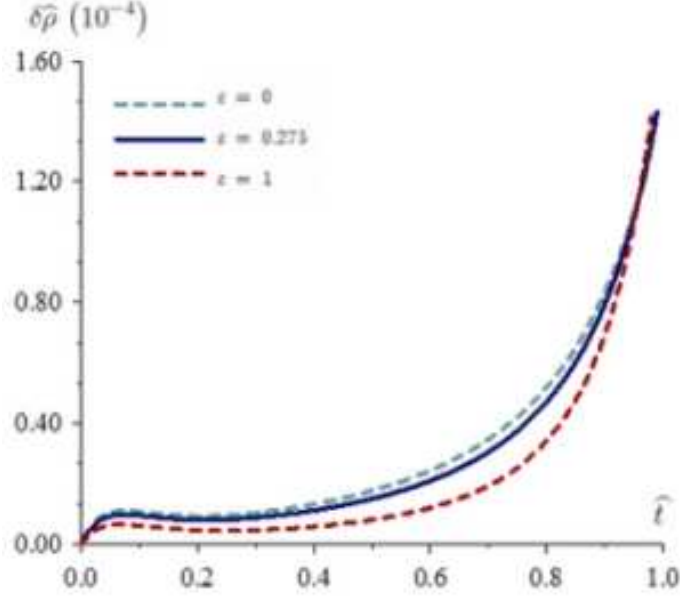


FIG. 4: Time evolution of the perturbation caused by the Hawking radiation in the brane energy density, for initial brane energy density $\hat{\rho}_0 = 300$ and transmission rates $\varepsilon = 0, 0.275$ and 1 .

We see that the evolution of the brane energy density is not very much affected by the transmission, rather it is just slightly rescaled.

Fig. 5, shows the time evolution of the perturbation caused by the bulk Hawking radiation in two of the source terms of the Friedmann equation. These are the asymmetry and the dark radiation terms (defined as in [24]). The two terms are roughly equal, but of opposite sign for a *critical* initial energy density $\hat{\rho}_0^{crit} = 520$, corresponding to the $\varepsilon = 0$ case, with the exceptions of the very early and very late stages of the evolution of the Friedmann brane [24]. By increasing the transmission, the evolutions will change drastically the late-time behaviour (Fig. 5). Furthermore, the changes induced at late times by transmission are significant, regardless of the particular value of the initial energy density (Fig. 6a is for light branes, while Fig. 6b for heavy branes).

At early times the transmission induces changes only for light branes, characterized by low initial energy density values (see Fig. 6a). We also remark that in the early stages of evolution of light branes a higher value of the transmission rate decreases the magnitude of the asymmetry source term, however the magnitude of the dark radiation source term is increased. This tendency is reversed in the late stages of evolution.

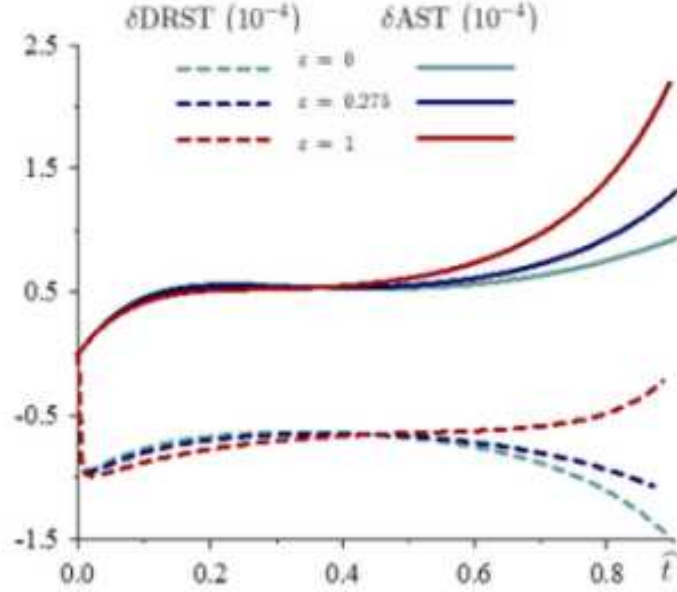


FIG. 5: Time evolution of the perturbations caused by the Hawking radiation in the source terms of the Friedmann equation. All solid lines represent perturbations in the *asymmetry source term*, for transmission rates $\varepsilon = 0, 0.275$ and 1 . All dotted lines represent perturbations in the *dark radiation source term*, for the same transmission rates. The plot is for the initial brane energy density $\hat{\rho}_0 = 520$ (the critical value of the brane initial energy density for opaque branes).

IV. CONCLUDING REMARKS

In this paper we have considered a highly asymmetric, closed brane-world model, with only one black hole in the bulk. The black hole is emitting Hawking radiation and this is partially transmitted through the brane to the other bulk region. The modifications to standard cosmological evolution caused by both the asymmetric setup and the radiation in the bulk represent small perturbations. By varying the transmission rate from total absorption (opaque brane) to total transmission we have seen, how the rate of transmission affects these perturbations. We have determined numerically the value $\varepsilon = 0.275$ of the transmission rate for which the critical-like behaviour discussed in [24] disappears. For all branes with a high degree of opacity the critical behaviour can be found for certain value of the initial energy density. The bigger the transmission rate, the lower this energy density and the lighter the brane with critical behaviour. For $\varepsilon = 0.275$ the critical initial energy density on the brane would be zero.

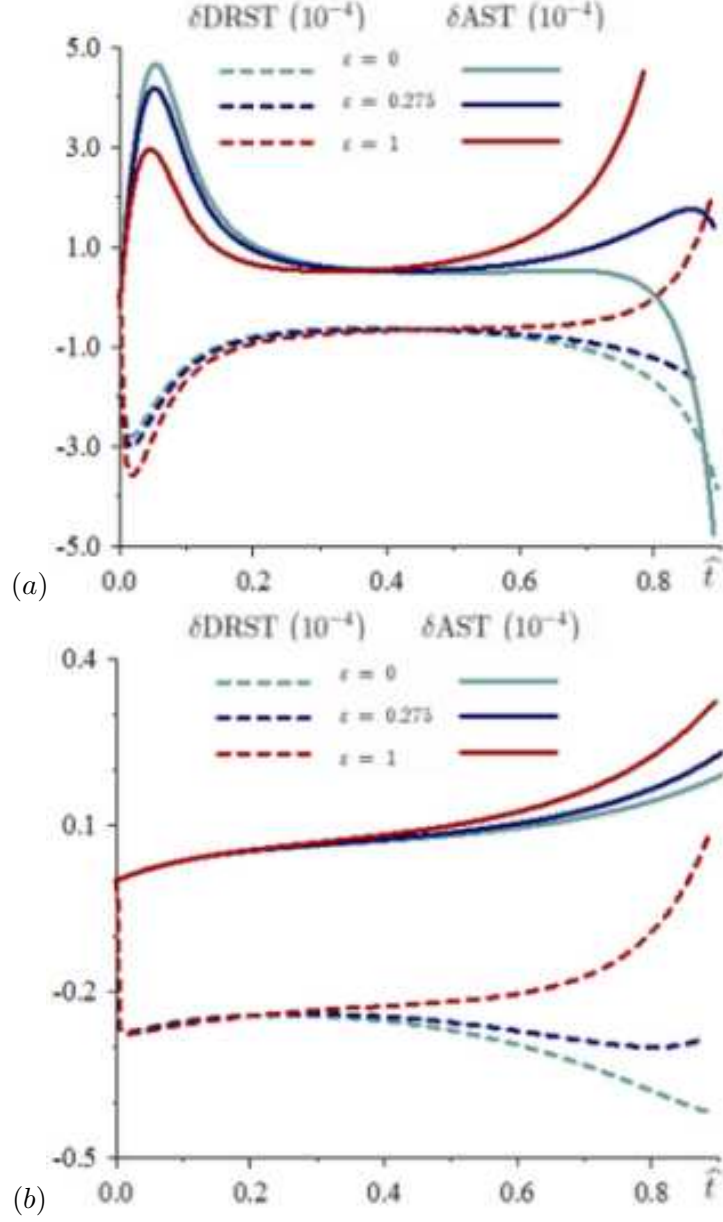


FIG. 6: As in Fig. 5, but for light and heavy branes, respectively. The plots are for the initial energy densities (a) $\hat{\rho}_0 = 100$ and (b) 2000.

We have also studied the evolution of the dark radiation and asymmetry source terms of the Friedmann equation. The evolution of these source terms was also discussed in [25], but in a different setup ($k = 0$, $\varepsilon = 1$, two black holes in the bulk and asymmetry in the cosmological constant). In that model the dark radiation term decreased, while the asymmetry term increased due to the black hole radiation. Our investigations lead to the same conclusion, however they also show the modulation of these effects due to the transmission rate. The

suppression of the dark radiation term is more accentuated for high ε at early times and for small ε at late times. By contrast the asymmetry term increases more significantly for small ε at early times and for high ε at late times.

As a generic rule we have found that the semi-transparent brane-worlds recollapse faster when the transmission rate is high. Thus the opaque branes discussed in [24] live the longest while the fastest recollapse occurs for total transmission.

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